What is the standard form equation of the ellipse that has vertices $(-3, 3)$ and $(5, 3)$ and foci $\left(1 - 2\sqrt{3}, 3\right)$ and $\left(1 + 2\sqrt{3}, 3\right)$?

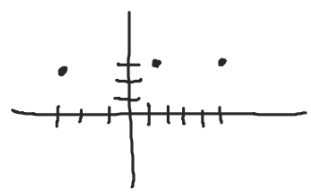
Center $(1, 3)$

$a = 4$

$b = ?$

$c = 2\sqrt{3}$

$$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{4} = 1$$



$c^2 = a^2 - b^2$

$(2\sqrt{3})^2 = 4^2 - b^2$

$12 = 16 - b^2$

$-4 = -b^2$

$b^2 = 4$

HOW TO

Given the standard form of an equation for an ellipse centered at $(0, 0)$, sketch the graph.

1. Use the standard forms of the equations of an ellipse to determine the major axis, vertices, co-vertices, and foci.

   a. If the equation is in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where $a > b$, then

      - the major axis is the *x*-axis
      - the coordinates of the vertices are $(\pm a, 0)$
      - the coordinates of the co-vertices are $(0, \pm b)$
      - the coordinates of the foci are $(\pm c, 0)$

   b. If the equation is in the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $a > b$, then

      - the major axis is the *y*-axis
      - the coordinates of the vertices are $(0, \pm a)$
      - the coordinates of the co-vertices are $(\pm b, 0)$
      - the coordinates of the foci are $(0, \pm c)$

2. Solve for c using the equation $c^2 = a^2 - b^2$.
3. Plot the center, vertices, co-vertices, and foci in the coordinate plane, and draw a smooth curve to form the ellipse.

Graphing an Ellipse Centered at the Origin

Graph the ellipse given by the equation, $\frac{x^2}{9} + \frac{y^2}{25} = 1$. Identify and label the center, vertices, co-vertices, and foci.

Graph the ellipse given by the equation $\frac{x^2}{36} + \frac{y^2}{4} = 1$. Identify and label the center, vertices, co-vertices, and foci.

Graphing an Ellipse Centered at the Origin from an Equation Not in Standard Form

Graph the ellipse given by the equation $4x^2 + 25y^2 = 100$. Rewrite the equation in standard form. Then identify and label the center, vertices, co-vertices, and foci.

$$\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

$$4x^2 + 16y^2 = 1$$

Graph the ellipse given by the equation $49x^2 + 16y^2 = 784$. Rewrite the equation in standard form. Then identify and label the center, vertices, co-vertices, and foci.

$$\frac{49x^2}{784} + \frac{16y^2}{784} = \frac{784}{784}$$

$$\frac{x^2}{16} + \frac{y^2}{49} = 1$$

HOW TO

Given the standard form of an equation for an ellipse centered at (h, k) , sketch the graph.

1. Use the standard forms of the equations of an ellipse to determine the center, position of the major axis, vertices, co-vertices, and foci.

   a. If the equation is in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where $a > b$, then

      - the center is (h, k)
      - the major axis is parallel to the *x*-axis
      - the coordinates of the vertices are $(h \pm a, k)$
      - the coordinates of the co-vertices are $(h, k \pm b)$
      - the coordinates of the foci are $(h \pm c, k)$

   b. If the equation is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$, where $a > b$, then

      - the center is (h, k)
      - the major axis is parallel to the *y*-axis
      - the coordinates of the vertices are $(h, k \pm a)$
      - the coordinates of the co-vertices are $(h \pm b, k)$
      - the coordinates of the foci are $(h, k \pm c)$

2. Solve for c using the equation $c^2 = a^2 - b^2$.
3. Plot the center, vertices, co-vertices, and foci in the coordinate plane, and draw a smooth curve to form the ellipse.

Graphing an Ellipse Centered at (*h*, *k*)

Graph the ellipse given by the equation, $\frac{(x+2)^2}{4} + \frac{(y-5)^2}{9} = 1$. Identify and label the center, vertices, co-vertices, and foci.

Graph the ellipse given by the equation $\frac{(x-4)^2}{36} + \frac{(y-2)^2}{20} = 1$. Identify and label the center, vertices, co-vertices, and foci.

HOW TO

Given the general form of an equation for an ellipse centered at (*h*, *k*), express the equation in standard form.

1. Recognize that an ellipse described by an equation in the form $ax^2 + by^2 + cx + dy + e = 0$ is in general form.
2. Rearrange the equation by grouping terms that contain the same variable. Move the constant term to the opposite side of the equation.
3. Factor out the coefficients of the x^2 and y^2 terms in preparation for completing the square.
4. Complete the square for each variable to rewrite the equation in the form of the sum of multiples of two binomials squared set equal to a constant, $m_1(x-h)^2 + m_2(y-k)^2 = m_3$, where m_1, m_2, and m_3 are constants.
5. Divide both sides of the equation by the constant term to express the equation in standard form.

Graphing an Ellipse Centered at (*h*, *k*) by First Writing It in Standard Form

Graph the ellipse given by the equation $4x^2 + 9y^2 - 40x + 36y + 100 = 0$. Identify and label the center, vertices, co-vertices, and foci.

$$4x^2 - 40x \quad + 9y^2 + 36y \quad = -100$$

$$4(x^2 - 10x + 25) + 9(y^2 + 4y + 4) = -100 + 100 + 36$$

$$\frac{4(x-5)^2}{36} + \frac{9(y+2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-5)^2}{9} + \frac{(y+2)^2}{4} = 1$$

Express the equation of the ellipse given in standard form. Identify the center, vertices, co-vertices, and foci of the ellipse. The graph the equation.

$$4x^2 + y^2 - 24x + 2y + 21 = 0$$

$$4x^2 - 24x \quad + y^2 + 2y \quad = -21$$

$$4(x^2 - 6x + 9) + (y^2 + 2y + 1) = -21 + 36 + 1$$

$$\frac{4(x-3)^2}{16} + \frac{(y+1)^2}{16} = \frac{16}{16}$$

$$\frac{(x-3)^2}{4} + \frac{(y+1)^2}{16} = 1$$

$$4x^2 - 24x \quad + 36y^2 - 360y \quad = -864$$

$$4(x^2 - 6x + 9) + 36(y^2 - 10y + 25) = -864 + 36 + 900$$

$$\frac{4(x-3)^2}{72} + \frac{36(y-5)^2}{72} = \frac{72}{72}$$

$$\frac{(x-3)^2}{18} + \frac{(y-5)^2}{2} = 1$$

$$4(x+3)^2 + 25(y-2)^2 =$$

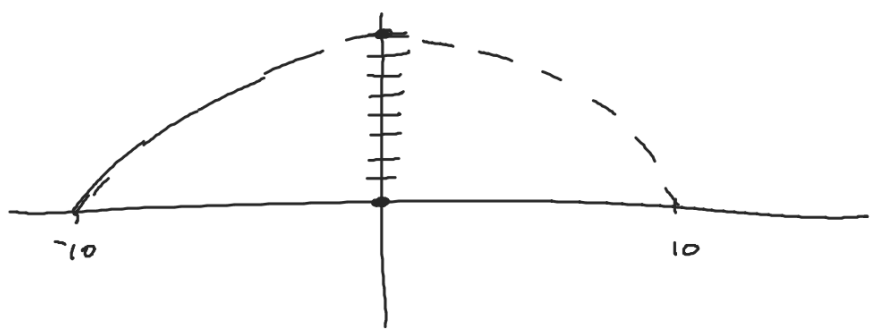
$$\frac{(x+3)^2}{9} + \frac{25(y-2)^2}{36}$$

$$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{\frac{36}{25}} = 1$$

$$x^2 + 2x \qquad 100y^2 - 1000y \qquad = -2401 + 1 + 2500$$

$$(x^2 + 2x + 1) + 100(y^2 - 10y + 25) = 100$$

$$\frac{(x+1)^2}{100} + \frac{(y-5)^2}{1} = 1$$



$$\frac{x^2}{100} + \frac{y^2}{64} = 1$$

$$\frac{16}{100} + \frac{y^2}{64} = 1$$

$$\frac{y^2}{64} = \frac{84}{100}$$

$$y^2 = \frac{5376}{100}$$

$$= 53.76$$

$$= 7.33$$